ELL 319: Digital Signal Processing (Fall 2015)

Minor Exam # 2

Time allowed: 60 mins

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1. Let $x[n]$ be a real-valued minimum phase sequence, then is $y[n] = (-1)^n x[n]$ a minimum phase sequence? Justify your answer. (15 points)

$Y(z) = X(-z)$ as $x[n] \xleftrightarrow{Z} X(z)$

$z_0^n x[n] \xleftrightarrow{Z} X(z/z_0)$

Since $x[n]$ is minimum phase sequence.

all poles and zeroes of $X(z)$ are inside unit circle.

If z_0 is a zero of $X(z)$ then $z = -z_0$ is a zero of $Y(z)$

If z_0 is a pole of $X(z)$ then $z = -z_0$ is a pole of $Y(z)$

$|z_0| < 1$ (∵ $X(z)$ is min-phase)

$\Rightarrow |-z_0| < 1 \Rightarrow Y(z)$ is min-phase seq.

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2. Determine the coefficients of a linear-phase FIR filter $y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2]$ such that it rejects completely a frequency component at $\omega_0 = 2\pi/3$ and its frequency response is normalized so that $H(e^{j\omega}) = 1$ at $\omega = 0$. (15 points)

$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z)$

$H(z) = \dfrac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2}$

$\pi \cdot \frac{\pi}{3}$

Let $\omega = e^{-j4\pi/3}$

$\omega^2 = e^{-j2\pi/3}$

$1, \omega, \omega^2 \rightarrow$ complex cube root of unity.

$1 + \omega + \omega^2 = 0$

$H(e^{j\omega}) = b_0 + b_1 e^{-j\omega} + b_2 e^{-2j\omega}$

$H(e^{j2\pi/3}) = b_0 + b_1 e^{-j2\pi/3} + b_2 e^{-j4\pi/3} = 0$

$H(e^{j\omega})\Big|_{\omega=0} = 1 \Rightarrow H(e^{j0}) = 1 \Rightarrow H(1) = 1$

$b_0 + b_1 + b_2 = 1 \Rightarrow b_0 + 2b_1 = 1$ (Put $b_2 = b_1$)

$b_0 = 1 - 2b_1$

$b_0 + b_1\omega^2 + b_2\omega = 0$

$b_0 + b_1(-1-\omega) + b_2\omega = 0$

$(b_0 - b_1) + (b_2 - b_1)\omega = 0$

$(b_0 - b_1) + (b_2 - b_1)\left[\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right] = 0 \Rightarrow b_2 - b_1 = 0$

$b_2 = b_1$

$$\boxed{\begin{array}{l} (1 - 3b_1) + 0 = 0 \\ \Rightarrow b_1 = 1/3 \\ b_2 = 1/3 \\ b_0 = 1 - 2/3 = 1/3 \end{array}}$$

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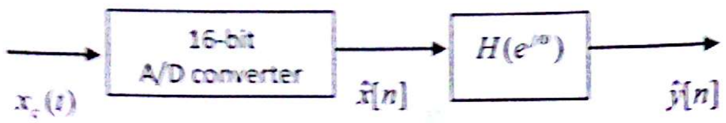
3. Consider the system shown below:



(25 points)

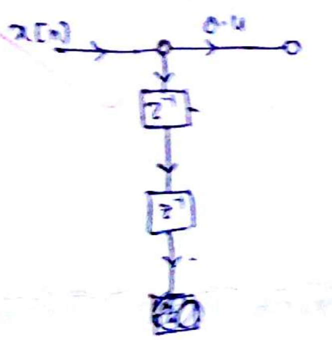
Impulse response of the digital filter is: $h[n] = 0.4\delta[n] + 0.8\delta[n-1] + 0.4\delta[n-2]$. Assume that the filter is implemented with 16-bit fixed point arithmetic (using two's complement) and the products are rounded to 16-bits before being accumulated to produce the output. Use the linear noise model to analyse this system.

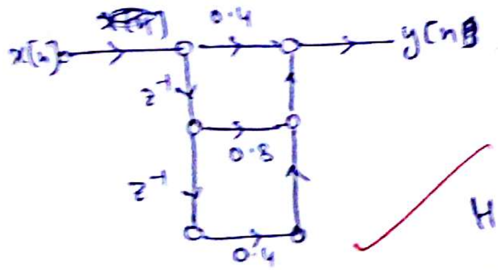
(a) Determine the maximum magnitude of $\hat{x}[n]$ such that no overflow can possibly occur in the implementing the digital filter.

(b) Draw a detailed linear noise model for the complete system (including the A/D converter).

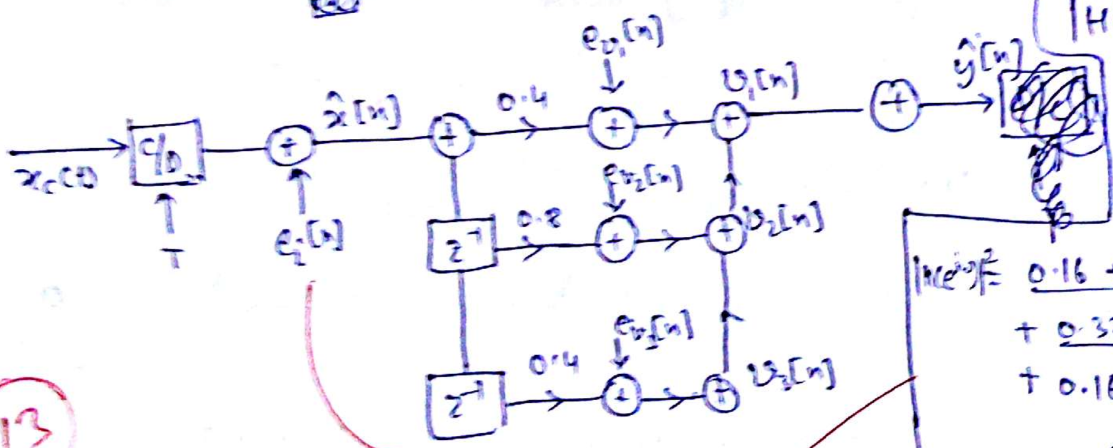
(c) Determine the total noise power at the output σ_f^2. (You can express the result in terms of σ_B^2)

$$H(z) = 0.4 + 0.8z^{-1} + 0.4z^{-2}$$





$$H(e^{j\omega}) = 0.4 + 0.8e^{-j\omega} + 0.4e^{-2j\omega}$$



$$|H(e^{j\omega})|^2 = (0.4 + 0.8e^{-j\omega} + 0.4e^{-2j\omega})(0.4 + 0.8e^{j\omega} + 0.4e^{2j\omega})$$

$$|H(e^{j\omega})|^2 = 0.16 + 0.32e^{j\omega} + 0.16e^{2j\omega} + 0.32e^{-j\omega} + 0.64 + 0.32e^{j\omega} + 0.16e^{-2j\omega} + 0.32e^{-j\omega} + 0.16$$

$$= 0.96 + 0.64(e^{j\omega} + e^{-j\omega}) + 0.16(e^{2j\omega} + e^{-2j\omega})$$

$$= 0.96 + 0.64(2\cos\omega) + 0.32\cos(2\omega)$$

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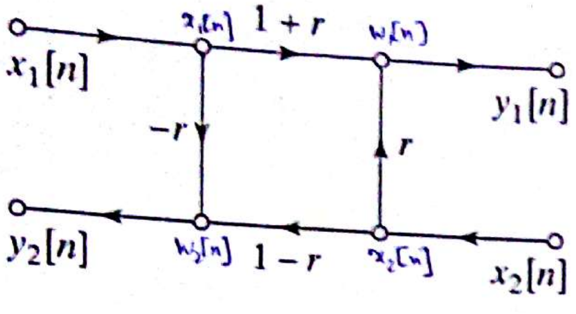
$$\sigma_f^2 = \frac{3\sigma_B^2}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 \, d\omega$$

$$= \frac{3\sigma_B^2}{2\pi} \times \cancel{0.96}\ 0.48 + 3\sigma_B^2$$

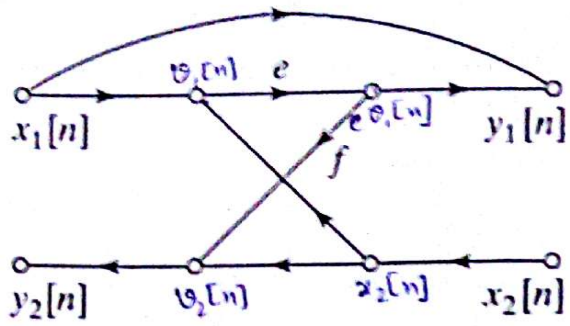
$$\boxed{\sigma_f^2 = \frac{1.44\sigma_B^2}{\pi}}$$

use Parseval's relation

4. Consider the two systems A and B shown below, each of which is a two-input, two-output system:



System A



System B

(a) Determine the difference equation for System A. (20 points)

(b) Determine the values of e and f for System B in terms of r such that the two systems are equivalent.

(c) Which implementation structure might be preferred? Give justification for your answer.

(a)

$w_1[n] = (1+r)\,x_1[n] + r\,x_2[n]$ $\quad w_1[n] = y_1[n]$

$w_2[n] = (1-r)\,x_2[n] - r\,x_1[n]$ $\quad y_2[n] = w_2[n]$

$y_1[n] = (1+r)\,x_1[n] + r\,x_2[n]$

$y_2[n] = -r\,x_1[n] + (1-r)\,x_2[n]$

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(b)

$v_1[n] = x_1[n] + x_2[n]$

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$v_2[n] = ef\,v_1[n] + x_2[n]$

$y_1[n] = x_1[n] + e v_1[n]$

$= x_1[n] + e\,x_1[n] + e\,x_2[n]$

$y_1[n] = \cancel{x_0}\,(1+e)\,x_1[n] + e\,x_2[n]$ $\quad \boxed{e = r}$ (By comparison)

$y_2[n] = v_2[n] = ef\,x_1[n] + \cancel{e}\,ef\,x_2[n] + x_2$

$y_2[n] = ef\,x_1[n] + (ef+1)\,x_2[n]$

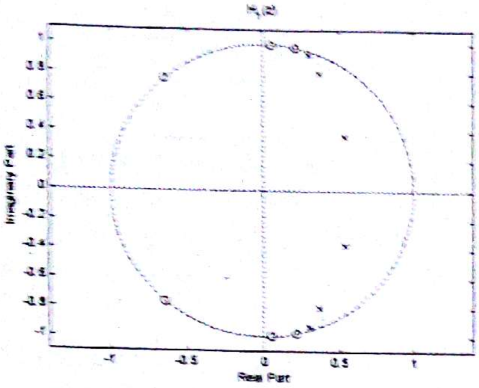
$ef = -r$ (By comparison)

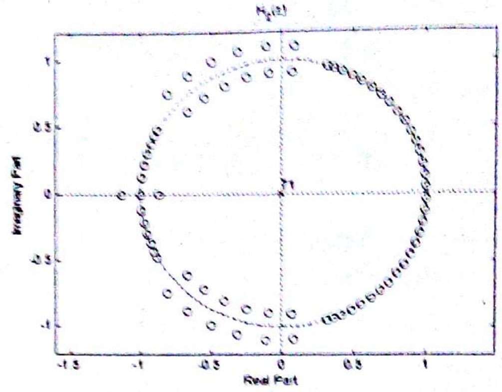
$\boxed{f = -1}$

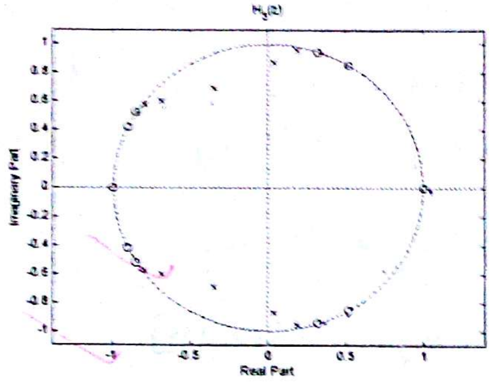
(c)

5. Consider the following pole-zero plots of three digital filters:

(25 points)







~~FIR~~

(a) FIR ~~X~~ poles only at $z=0$ & no where else.

So $H_2(z)$: FIR

$H_1(z), H_3(z)$: IIR

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For each of the three filters, determine the following:
(a) Filter is FIR or IIR, (b) Filter order,
(c) Filter is low-pass (LP), high-pass (HP), band-pass (BP), or band-stop (BS), and
(d) Approximate cutoff or edge frequencies for each filter. Brief explanation is required for full credit.

$H_1(z)$: 6 poles and 6 zeroes [poles and zeroes occur in complex conjugate pairs]

(b) Order of filter: $H_1(z) \rightarrow 6$ $H_2(z) \rightarrow 71$ $H_3(z) \rightarrow 10$
= max{no. of poles, zeros}

(c) Type of Filter: ~~$|H(e^{j\omega})|$ is allowed only for right half~~

$H_1(z)$: low-pass

$H_2(z)$: low-pass

$H_3(z)$: ~~Band pass~~ ~~Band~~ High-pass (does not allow freq → 0. zero at $z=1$)

(d) ?